Introduction to RF Simulation and Its Application

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***Abstract* — RF circuits exhibit several distinguishing characteristics that make them difficult to simulate using traditional SPICE transient analysis. The various extensions to the harmonic balance and shooting method simulation algorithms are able to exploit these characteristics to provide rapid and accurate simulation for these circuits.**

This paper is an overview of RF simulation beginning with the characteristics of RF circuits that distinguish them from traditional analog circuits. Harmonic balance and shooting methods are described along with several important extensions. The emphasis is on presenting the algorithms at a conceptual level to provide a basic understanding of the operation and capabilities of the algorithms. Finally, it is shown how these techniques are applied to make common RF measurements.

I. Introduction

The increasing demand for low-cost mobile communication systems has greatly expanded the need for simulation algorithms that are both efficient and accurate when applied to RF communication circuits.

This paper is an introduction to RF simulation methods and how they are applied to make common RF measurements. It describes the unique characteristics of RF circuits, the methods developed to simulate these circuits, and the application of these methods. See [15] for a recent survey of RF simulation methods.

II. Wireless Communication

In wireless communication, signals are sent through the ether between two transceivers, a transceiver being a combination of a transmitter and a receiver. To avoid interference, each pair of transceivers in a particular location are assigned a band of frequencies over which they communicate. The job of a transceiver is to translate the signals to be communicated to and from the assigned band. They must pass the communications faithfully despite the shortcomings of the ether as a channel. In particular, signals generally exhibit high attenuation and are subject to large interfering signals.

A. Small Desired Signals

Receivers must be very sensitive to detect small input signals. Typically, receivers are expected to operate with as little as 1 μV at the input. The sensitivity of a receiver is limited by the noise generated in the input circuitry of the receiver. Thus, noise is a big concern in receivers and the ability to simulate noise is very important. As shown in Figure 1, a typical superheterodyne receiver first filters and then amplifies its input with a low noise amplifier or LNA. It then translates the signal to the intermediate frequency or IF by mixing it with the first local oscillator or LO. The noise performance of the front-end is determined by the LNA, the mixer, and the LO. While it is possible to use traditional SPICE noise analysis to find the noise of the LNA, it is useless on the mixer and the LO because the noise in these blocks is strongly influenced by the large LO signal.

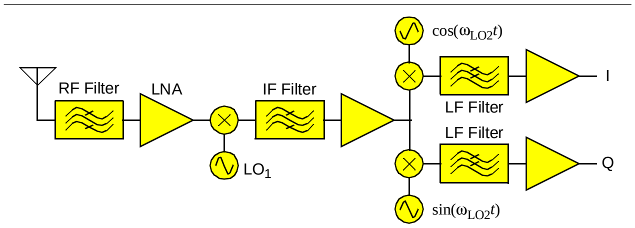

Fig. 1. A coherent superheterodyne receiver.

The small input signal levels requires that receivers must be capable of a tremendous amount of amplification. Often as much as 120 dB of gain is needed. With such high gain, any coupling from the output back to the input can cause problems. One important reason why the superheterodyne receiver architecture is used is to spread that gain over several frequencies to reduce the chance of coupling. It also results in the first LO being at a different frequency than the input, which prevents this large signal from contaminating the small input signal. For various reasons, the direct conversion or homodyne architecture is a candidate to replace the superheterodyne architecture in some wireless communication systems [1,24,25]. In this architecture the RF input signal is directly converted to baseband in one step. Thus, most of the gain will be at baseband and the LO will be at the same frequency as the input signal. In this case, the ability to verify design in the face of small amounts of coupling is quite important and will require careful modeling of the significant stray signal paths, such as coupling through the substrate, between package pins and bondwires, and through the supply lines.

B. *Large Interfering Signals*

Receivers must be sensitive to small signals even in the presence of large interfering signals. This situation arises when trying to receive a weak or distant transmitter with a strong nearby transmitter broadcasting in an adjacent channel. The interfering signal can be 60-70 dB larger than the desired signal and can act to block its reception by overloading the input stages of the receiver or by increasing the amount of noise generated in the input stage. Both of these problems result if the input stage is driven into a nonlinear region by the interferer. To avoid these problems, the front-end of a receiver must be very linear. Thus, linearity is also a big concern in receivers. Receivers are narrowband circuits and so the nonlinearity is quantified by measuring the intermodulation distortion. This involves driving the input with two sinusoids that are in band and close to each other in frequency and then measuring the intermodulation products. This is generally an expensive simulation with SPICE because many cycles must be computed in order to have the frequency resolution necessary to see the distortion products.

Distortion also plays an important role in the transmitter where nonlinearity in the output stages can cause the bandwidth of the transmitted signal to spread out into adjacent channels. This is referred to as spectral regrowth because, as shown in Figure 2, the bandwidth of the signal is limited before it reaches the transmitter’s power amplifier or PA, and distortion in the PA causes the bandwidth to increase again. If it increases too much, the transmitter will not meet its adjacent channel power requirements (ACPR). When transmitting digitally modulated signals, spectral regrowth is virtually impossible to predict with SPICE. The transmission of at least 1000 digital symbols must be simulated to get a representative spectrum, and this combined with the high carrier frequency makes use of transient analysis impractical.

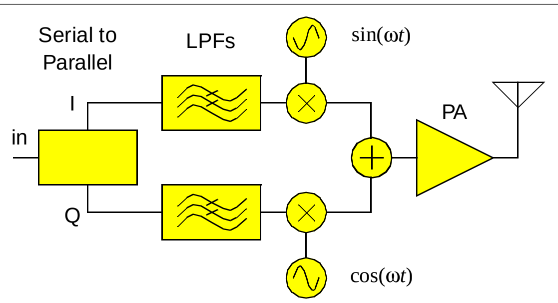

Fig. 2. A digital direct conversion transmitter.

III. CHARACTERISTICS OF RF CIRCUITS

RF circuits have several unique characteristics that are barriers to the application of traditional circuit simulation techniques. Over the last decade, researchers have developed many special purpose algorithms that overcome these barriers to provide practical simulation for RF circuits, often by exploiting the very characteristic that represented the barrier to traditional methods.

A. *Narrowband Signals*

RF circuits process narrowband signals in the form of modulated carriers. Modulated carriers are characterized as having a periodic high-frequency carrier signal and a low-frequency modulation signal that acts on either the amplitude, phase, or frequency of the carrier. For example, a typical cellular telephone transmission has a 10-30 kHz modulation bandwidth riding on a 1-2 GHz carrier. In general, the modulation is arbitrary, though it is common to use simple periodic or quasiperiodic modulations constructed from a small number of sinusoids for test signals.

The ratio between lowest frequency present in the modulation and the frequency of the carrier is a measure of the relative frequency resolution required of the simulation. General purpose circuit simulators, such as SPICE, use transient analysis to predict the nonlinear behavior of a circuit. Transient analysis is inefficient when it is necessary to resolve low modulation frequencies in the presence of a high carrier frequency because the high-frequency carrier forces a small time step while a low-frequency modulation forces a long simulation interval.

Passing a narrowband signal though a nonlinear circuit results in a broadband signal whose spectrum is relatively sparse, as shown in Figure 3. In general, this spectrum consists of clusters of frequencies near the harmonics of the carrier. These clusters take the form of a discrete set of frequencies if the modulation is periodic or quasiperiodic, and a continuous distribution of frequencies otherwise.

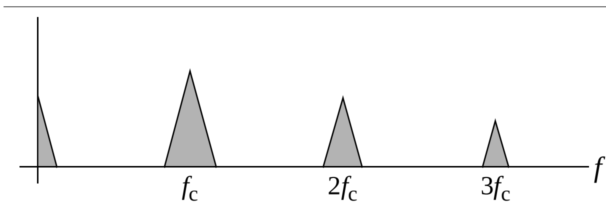

Fig. 3. Spectrum of a narrowband signal centered at a carrier frequency f_c after passing though a nonlinear circuit.

RF simulators exploit the "sparse" nature of this spectrum in various ways and with varying degrees of success. Steady-state methods are used when the spectrum is discrete, and transient methods are used when the spectrum is continuous.

B. Time-Varying Linear Nature of the RF Signal Path

Another important but less appreciated aspect of RF circuits is that they are generally designed to be as linear as possible from input to output to prevent distortion of the *modulation* or *information signal*. Some circuits, such as mixers, are designed to translate signals from one frequency to another. To do so, they are driven by an additional signal, the LO, a large periodic signal the frequency of which equals the amount of frequency translation desired. For best performance, mixers are designed to respond in a strongly nonlinear fashion to the LO. Thus, mixers behave both near-linearly (to the input) and strongly nonlinearly (to the LO).

Since *timing* or *synchronization signals*, such as the LO or the clock, are not part of the path of the information signal, they may be considered to be part of the circuit rather than an input to the circuit as shown in Figure 4.

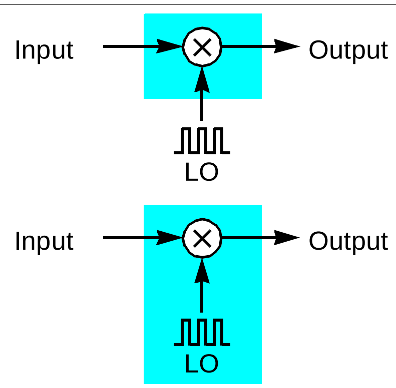

Fig. 4. One can generally approximate a nonlinear periodically-driven circuit (above) with a linear periodically-varying circuit (below).

This simple change of perspective allows the mixer to be treated as having a single input and a near-linear, though periodically time-varying, transfer function. As an example, consider a mixer made from an ideal multiplier and followed by a low-pass filter. A multiplier is nonlinear and has two inputs. Applying an LO signal of $\cos(\omega_{LO} t)$ consumes one input and results in a transfer function of

$$v_{out}(t) = \mathrm{LPF}\{\cos(\omega_{LO} t) v_{in}(t)\}, \tag{1}$$

which is clearly time-varying and is easily shown to be linear with respect to v_{in}. If the input signal is

$$v_{in}(t) = m(t)\cos(\omega_c t), \tag{2}$$

then

$$v_{out}(t) = \mathrm{LPF}\{m(t)\cos(\omega_c t)\cos(\omega_{LO} t)\} \tag{3}$$

and

$$v_{out}(t) = m(t)\cos((\omega_c - \omega_{LO})t). \tag{4}$$

This demonstrates that a linear periodically-varying transfer function implements frequency translation.

Often we can assume that the information signal is small enough to allow the use of a linear approximation of the circuit from its input to its output. Thus, a small-signal analysis can be performed, as long as it accounts for the periodically varying nature of the signal path, which is done by linearizing about the periodic operating point. This is the idea behind the small-signal analyses of Section VI. Traditional simulators such as SPICE provide several small-signal analyses, such as the AC and noise analyses, that are considered essential when analyzing amplifiers and filters. However, they start by linearizing a nonlinear time-invariant circuit about a constant operating point, and so generate a linear time-invariant representation, which cannot exhibit frequency translation. By linearizing a nonlinear circuit about a periodically varying operating point, we extend small-signal analysis to circuits that must have a periodic timing signal present to operate properly, such as mixers, switched filters, samplers, and oscillators (for oscillators the timing signal is the desired output of the oscillator, while the information signal is generally an undesired signal, such as the noise). In doing so, a periodically varying linear representation results, which does exhibit frequency translation.

All of the traditional small-signal analyses can be extended in this manner, enabling a wide variety of applications (some of which are described in [33]). In particular, a noise analysis that accounts for noise folding and cyclostationary noise sources can be implemented [20,28], which fills a critically important need for RF circuits. When applied to oscillators, it also accounts for phase noise [3,4,10,11].

C. Linear Passive Components

At the high frequencies present in RF circuits, the passive components, such as transmission lines, spiral inductors, packages (including bond wires) and substrates, often play a significant role in the behavior of the circuit. The nature of such components often make them difficult to include in the simulation.

Generally the passive components are linear and are modeled with phasors in the frequency-domain, using either analytical expressions or tables of S-parameters. This greatly simplifies the modeling of distributed components such as transmission lines. Large distributed structures, such as packages, spirals, and substrates, often interface with the rest of the circuit through a small number of ports. Thus, they can be easily replaced by a N-port macromodel that consists of the N^2 transfer functions. These transfer functions are found by reducing the large systems of equations that describe these structures, leaving only the equations that relate the signals at their ports. The relatively expensive reduction step is done once for each frequency as a preprocessing step. The resulting model is one that is efficient to evaluate in a frequency-domain simulator if N is small. This is usually true for transmission lines and spirals, and less often true for packages and substrates.

Time-domain simulators are formulated to solve sets of first-order ordinary-differential equations (ODE). However, distributed components, such as transmission lines, are described with partial-differential equations (PDE) and so are problematic for time-domain simulators. Generally, the PDEs are converted to a set of ODEs using some form of discretization [17]. Such approaches suffer from bandwidth limits. A alternative approach is to compute the impulse response for a distributed component from a frequency domain description and use convolution to determine the response of the component in the circuit [9,30]. Evaluating lossy or dispersive transmission line models or tables of S-parameters with this approach is generally expensive and error-prone [31]. Packages, substrates and spirals can be modeled with large lumped networks, but such systems can be too large to be efficiently incorporated into a time domain simulation, and so some form of reduction is necessary [5,22].

IV. Basic RF Building Blocks

RF systems are constructed primarily using four basic building blocks — amplifiers, filters, mixers, and oscillators. Amplifiers and filters are common analog blocks and are well handled by SPICE. However, mixers and oscillators are not heavily used in analog circuits and SPICE has limited ability to analyze them. What makes these blocks unique is presented next.

A. Mixers

Mixers translate signals from one frequency range to another. They have two inputs and one output. One input is for the information signal and the other is for the timing signal, the LO. Ideally, the signal at the output is the same as that at the information signal input, except shifted in frequency by an amount equal to the frequency of the LO. As shown in Section III-B, a multiplier can act as a mixer. In fact, a multiplier is a reasonable model for a mixer except that the LO is generally passed through a limiter to make the output less sensitive to noise on the LO. Generally, the limiter is an integral part of the mixer.

The input and output signals of a mixer used for up-conversion (as in a transmitter) are shown in Figure 5. The LO

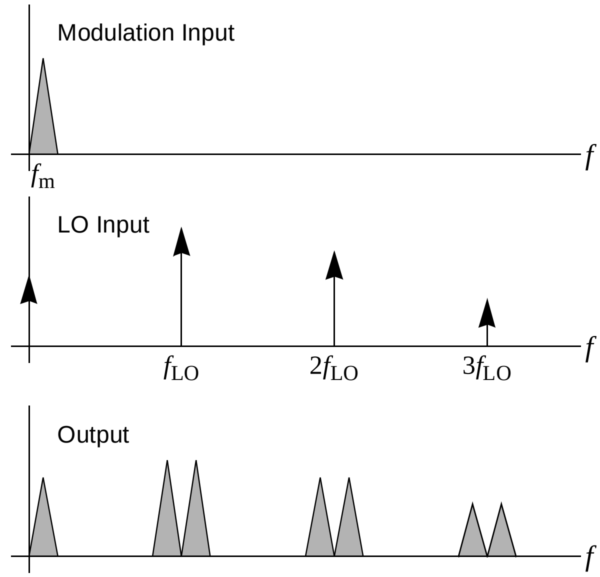

Fig. 5. Signals at the inputs and outputs of an up-conversion mixer. The modulation signal is mixed up to the upper and lower sidebands of the LO and its harmonics.

is shown after passing through the limiter so that the output in the time-domain is simply the product of the inputs, or the convolution of the two inputs in the frequency domain. The information signal, here a modulation signal, is replicated at the output above and below each harmonic of the LO. These bands of signal above and below each harmonic are referred to as *sidebands.* There are two sidebands associated with each harmonic of the LO. The ones above the harmonic are referred to as the *upper sidebands* and the ones below are referred to as the *lower sidebands*. The sideband at DC is referred to as the *baseband*. The size of each sideband is determined by the size of its associated harmonic.

When the LO has a rich harmonic content, an input signal at any sideband will be replicated to each of the sidebands at the output. To select the desired sideband, the mixer is followed by a filter.

Consider a down-conversion mixer (as in a receiver) and assume the mixer is followed by a filter. This filter is used to remove all but the desired channel. The output of the mixer/filter pair is sensitive to signals in each sideband of the LO. Associated with each sideband is a transfer function from that sideband to the output. The shape of the transfer function is determined largely by the filter. Thus, the bandwidth of the passband is that of the filter. If the filter is a bandpass, then the passband of the transfer function will be offset from the LO or its harmonic by the center frequency of the filter. These passbands are referred to as the *images* of the filter and are shown in Figure 6.

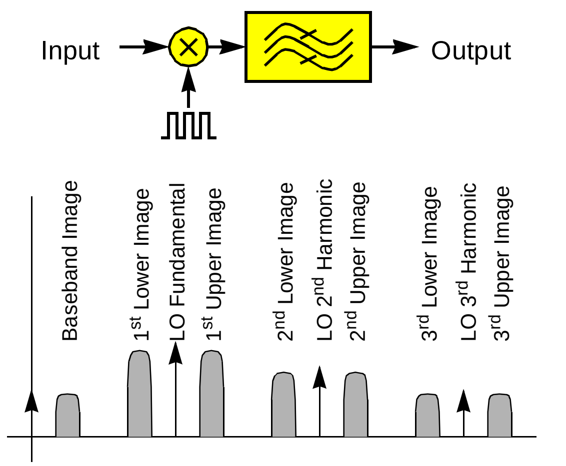

Fig. 6. Images at the input of the first mixing stage of a typical receiver. The images are frequency bands where the output is sensitive to signals at the input.

B. Oscillators

Oscillators generate a reference signal at a particular frequency. For example, they are used to generate the LO for mixers. In some oscillators, referred to as VCOs for voltage controlled oscillators, the frequency of the output varies proportionally to some input signal.

Compared to mixers, oscillators seem quite simple. That is an illusion.

Consider the trajectory of an oscillator's stable periodic orbit in state space. Furthermore, consider disturbing the oscillator by applying an impulse $u(t) = \delta(t)$. The oscillator responds by following a perturbed trajectory $v(t) + \Delta v(t)$ as shown in Figure 7, where $v(t)$ represents the unperturbed solution and $\Delta v(t)$ is the perturbation in the response.

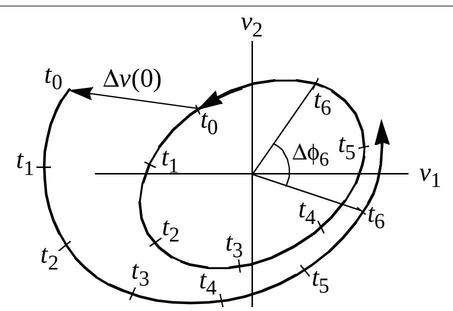

Fig. 7. The trajectory of an oscillator shown in state space with and without a perturbation Δv. By observing the time stamps $(t_0, ..., t_6)$ one can see that the deviation in amplitude dissipates while the deviation in phase does not.

Decompose the perturbed response into amplitude and phase variations.

$$v(t) + \Delta v(t) = (1 + \alpha(t))v\left(t + \frac{\phi(t)}{2\pi f_c}\right) \tag{5}$$

where $\alpha(t)$ represents the variation in amplitude, $\phi(t)$ is the variation in phase, and f_c is the oscillation frequency.

Since the oscillator is stable and the duration of the disturbance is finite, the deviation in amplitude eventually decays away and the oscillator returns to its stable orbit. In effect, there is a restoring force that tends to act against amplitude noise.

However, since the oscillator is autonomous, any time-shifted version of the solution is also a solution. Once the phase has shifted due to a perturbation, the oscillator continues on as if never disturbed except for the shift in the phase of the oscillation. There is no restoring force on the phase and so phase deviations accumulate.

After being disturbed by an impulse, the asymptotic response of the amplitude deviation is $\alpha(t) \to 0$ as $t \to \infty$. However, the asymptotic response of the phase deviation is $\phi(t) \to \Delta\phi$. If responses that decay away are neglected then the impulse response of the phase deviation $\phi(t)$ can be approximated with a unit step $s(t)$. Thus, the phase shift over time for an arbitrary u is

$$\phi(t) \sim \int_{-\infty}^{\infty} s(t-\tau)u(\tau)d\tau = \int_{-\infty}^{t} u(\tau)d\tau \tag{6}$$

or the power spectral density (PSD) of the phase is

$$S_\phi(f) \sim \frac{S_u(f)}{(2\pi f)^2}. \tag{7}$$

The disturbance u may be either deterministic or random in character and my result from extraneous signals coupling into the oscillator or from parametric variations in the components that make up the oscillator.

If $S_u(f)$ is white, then $S_\phi(f)$ is proportional to $1/(2\pi f)^2$. This result has been shown to apply at low frequencies, but with a more detailed derivation is can also be shown to be true over a broad range of frequencies [10]. Define a such that

$$S_\phi(f) = a\frac{f_c^2}{f^2} \tag{8}$$

where $f_c = 1/T$ is the carrier frequency. S_ϕ is the PSD of the phase variable in (5). Phase is not directly observable so instead one is often interested in the PSD of the signal Δv. Demir [4] shows that near the fundamental

$$S_{\Delta v}(f_c + f_m) = |V_1|^2 \frac{af_c^2}{a^2\pi^2 f_c^4 + f_m^2}, \tag{9}$$

where f_m is the frequency offset from the fundamental and V_1 is the first Fourier coefficient for v,

$$v(t) = \sum_{k=-\infty}^{\infty} V_k e^{j2\pi k f_c t}. \tag{10}$$

This spectrum is a Lorentzian with corner frequency $a\pi f_c^2$ and is shown in Figure 8. As $t \to \infty$ the phase of the oscil-

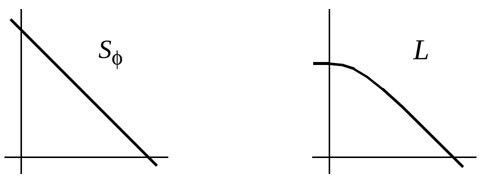

Fig. 8. Two different ways of characterizing phase noise in an oscillator. S_ϕ is the power spectral density (PSD) of the phase and L is PSD of the signal normalized to the power in the fundamental.

lator drifts without bound, and so $S_\phi(f_m) \to \infty$ as $f_m \to 0$. However, as $f_m \to 0$ the PSD of the signal $S_{\Delta v}(f_c + f_m) \to |X_1|^2/(a\pi^2 f_c^2)$, which is inversely proportional to a. Thus, the larger a, the more phase noise, the higher the corner frequency and the lower the low frequency noise level. This happens because the phase noise does not affect the total power in the signal, it only affects its distribution. Without phase noise, $S_{\Delta v}(f)$ is a series of impulse functions at the harmonics of f_c. With phase noise, the impulse functions spread, becoming fatter and shorter but retaining the same power [4].

It is more common to report phase noise as L, the ratio of the single-sideband (SSB) phase noise power to the power in the fundamental (in dBc / Hz)

$$L(f_m) = \frac{S_{\Delta v}(f_c + f_m)}{|V_1|^2} = \frac{a f_c^2}{a^2 \pi^2 f_c^4 + f_m^2}. \tag{11}$$

At frequencies above the corner $a\pi f_c^2$, the phase noise is approximated with[1]

$$L(f_m) = \frac{a f_c^2}{f_m^2} = S_\phi(f_m) \quad \text{for} \quad a\pi f_c^2 \ll f \ll f_c. \tag{12}$$

In the case where u represents flicker noise, $S_u(f)$ is generally pink or proportional to $1/f$. Then $S_\phi(f_m)$ would be proportional to $1/f^3$.

1. Many other references report that $L(f_m) = S_\phi(f_m)/2$, which is true when S_ϕ is the single-sided PSD [27,36]. In this paper, S_ϕ is the doubled-sided PSD.

V. Large Signal RF Simulation Techniques

Transient analysis, shooting methods, and harmonic balance represent the base methods from which RF simulation methods are constructed. They are introduced in this section with a brief discussion of their strengths and weaknesses. References are given to allow the methods to be studied in more depth [15].

A. Transient Analysis

Transient analysis breaks the time continuum into a series of adjacent short intervals and uses low-order polynomials to approximate the solution over each interval (the time step) with the constraint that the solution must be continuous across interval boundaries (the time points).

Transient analysis can be inefficient with modulated carrier circuits because the high frequency carrier requires a high density of points and the low frequency modulation will require a long simulation interval. Typically at least 20 timepoints are needed per cycle of the high frequency carrier and the carrier frequency will be up to a million times higher than the modulation frequency. The result is an extremely long simulation because of the number of timepoints that must be computed. In addition, long time constants in the circuit can require an even longer simulation interval.

B. Harmonic Balance

Harmonic balance [14,16] formulates the circuit equations and their solution in the frequency domain as a Fourier series. Fourier series cannot represent transient behavior, and so harmonic balance directly finds the steady-state solution. The linear device equations are actually formulated in the frequency domain using phasor analysis. However, this is generally not practical or desirable for the nonlinear device equations. Instead, the nonlinear devices are evaluated in the time domain. First the signals driving the nonlinear devices are converted from the frequency to the time domain using the inverse Fourier transform. The nonlinear devices are evaluated for a complete period of the time-domain waveform and the resulting response waveforms are converted back into the frequency domain using the Fourier transform.

An extremely important application of harmonic balance is determining the steady-state behavior of oscillators. To do so, it is necessary to modify harmonic balance to directly compute the operating frequency [14,26]. The Fourier transform is defined for periodic signals, however several methods have been developed to extend harmonic balance to handle quasiperiodic signals [14,37]. In addition, it has also be extended to allow the Fourier coefficients to be vary slowly with time [6,15,18,29]. These methods are referred to as *envelope* methods and are used to simulate modulated carrier signals where the modulation is not a repetitive signal.

The main strength of harmonic balance is its natural support for linear frequency-domain models. Distributed components such as lossy and dispersive transmission lines and interpolated tables of S-parameters from either measurements or electromagnetic simulators are examples of linear models that are handled easily and efficiently with harmonic balance.

Harmonic balance struggles on circuits that contain signals that exhibit sharp transitions, as is common in mixers and oscillators. In this case, a large number of frequencies is needed to accurately represent the signal, which increases the expense of harmonic balance. In addition, the magnitude of the harmonics drop slowly for signals with sharp transitions, making it difficult to know how many harmonics must be computed by harmonic balance. If too few harmonics are included, the results are inaccurate, if too many are included, the simulations are impractical. And often, increasing the number of harmonics include makes the simulation impractical before it makes it accurate.

C. Shooting Methods

Transient analysis solves initial-value problems. A shooting method is an iterative procedure layered on top of transient analysis that is designed to solve boundary-value problems. In the case of a periodic steady-state solution, the boundary condition is simple: the state of the circuit at the start of the period must be the same as at the end. With shooting methods, an initial state is chosen, the circuit is simulated for one cycle, and the final state is compared to the initial state. The initial state is adjusted and the procedure repeats until the initial and final states are the same [2,14,32,34].

As with harmonic balance, extensions exist that naturally handle autonomous [14], quasiperiodic [14], and transient modulated [13,21] carriers. The quasiperiodic shooting methods are referred to as *mixed frequency-time* methods and the transient shooting methods are referred to as *envelope following.*

Shooting methods are applied in the same situations as harmonic balance as long as the circuits do not include distributed components. It is generally preferred if the circuit is driven with strongly discontinuous signals (pulses as opposed to sinusoids). As such, shooting methods are well suited for simulating switching mixers, switched filters, samplers, frequency dividers, and relaxation oscillators.

VI. SMALL-SIGNAL RF SIMULATION TECHNIQUES

As pointed out in Section III-B, RF circuits can often be accurately modeled as linear periodically-varying circuits. Doing so is referred to periodic small-signal analysis because the input is assumed small enough so that it does not cause a nonlinear response. Periodic small-signal analysis provides significant advantages over trying to get the same information from a equivalent large signal analysis. First, they can be much faster. Second, a wider variety of analyses are available. For example, noise analysis is much easier to implement as a small signal analysis. Finally, they can be more accurate if the small signals are very small relative to the large signals. Small signals applied in a large signal analysis can be overwhelmed by errors that stem from the large signals. In a small signal analysis, the large and small signals are applied in different phases of the analysis. Errors in the large signal phase typically have only a minor affect on the linearization and hence the accuracy of the small signal results.

A great deal of useful information can be acquired by performing a small-signal analysis about the time-varying operating point of the circuit. Small-signal analyses start by performing the analyses described in the previous section to compute the periodic operating point with only the large timing or synchronization signals applied (the LO or the clock). The circuit is then linearized about this time-varying operating point and the small information signal applied. The response is calculated using linear time-varying analysis.

Consider a circuit whose input is the sum of two periodic signals, $u(t) = u_L(t) + u_s(t)$, where $u_L(t)$ is an arbitrary periodic waveform with period T_L and $u_s(t)$ is a sinusoidal waveform of radial frequency ω_s whose amplitude is small. In this case, $u_L(t)$ represents the timing signal and $u_s(t)$ represents the information signal.

Let $v_L(t)$ be the steady-state solution waveform when $u_s(t)$ is zero. Then allow $u_s(t)$ to be small, but nonzero. We can consider the new solution $v(t)$ to be a perturbation $v_s(t)$ on $v_L(t)$, as in $v(t) = v_L(t) + v_s(t)$. The small-signal solution $v_s(t)$ is computed by linearizing the circuit about $v_L(t)$ and applying one of the methods for finding the steady-state solution already described. From the theory of periodically time-varying systems [19,35], it is known that for

$$u_s(t) = U_s e^{j\omega_s t} \tag{13}$$

the steady-state response is given by

$$v_s(t) = \sum_{k=-\infty}^{\infty} V_s(k) e^{j(\omega_s + k\lambda)t} . \tag{14}$$

where $\lambda = 2\pi/T_L$ is the large signal fundamental frequency. $V_s(k)$ represents the sideband for the k^{th} harmonic of V_L. In this situation, shown in Figure 9, there is only one sideband per harmonic because U_s is a single frequency complex exponential and the circuit is linear. This representation has terms at negative frequencies. If these terms are mapped to positive frequencies, then the sidebands with $k < 0$ become lower sidebands of the harmonics of v_L and those with $k > 0$ become upper sidebands.

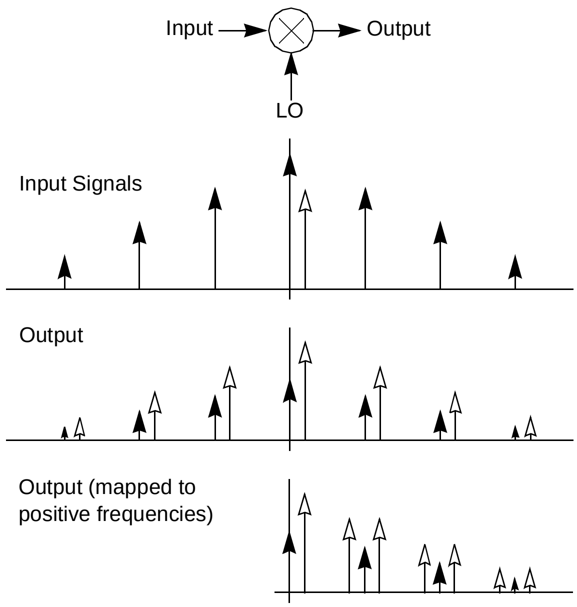

Fig. 9. The steady-state response of a linear periodically-varying system to a small complex exponential stimulus. The large signals are represented with solid arrows and the small signal with hollow arrows.

$V_s(k)/U_s$ is the transfer function for the input at ω_s to the output at $\omega_s + k\lambda$. Notice that with periodically-varying linear systems there are an infinite number of transfer functions between any particular input and output. Each represents a different frequency translation.

Extensions: Versions of this small signal analysis exists for both harmonic balance [8,12] and shooting methods [19,35]. They can be thought of as the extensions of the SPICE small-signal analyses to the situation where the circuit is linearized about a time-varying operating point. This is sufficient for performing a time-varying AC analysis and can be extended to other types of small-signal analyses, such as computing the S-parameters of the circuit. These small-signal analyses are also extendable to cyclostationary noise analysis [3,20,28], which is an extremely important capability for RF designers [33]. In addition, they can be used to predict the phase noise of oscillators [4,10,11]. They also have applications outside of RF circuits. For example, they can be applied to samplers, track-and-holds, switched-capacitor filters, frequency multipliers, frequency dividers, chopper stabilized amplifiers, etc. All of these circuits are periodically-driven near-linear signal processing circuits.

Cyclostationary Noise: With periodically-varying systems, there are two effects that act to complicate noise analysis. First, for noise sources that are bias dependent, such as shot noise sources, the time-varying operating point acts to modulate the noise sources. Such noise sources are referred to as being cyclostationary. Second, the transfer function from the noise source to the output is also periodically-varying and so acts to modulate the contribution of the noise source to the output.

Modulation is a multiplication of the signals in the time-domain and so in the frequency-domain the spectrum of the noise source is convolved with the spectrum of the transfer function. The transfer function is periodic and so has a discrete line spectrum. Convolution with a discrete spectrum involves a countable number of scale, shift, and sum operations, as shown in Figure 10. The final result is the sum of the noise contributions both up-converted and down-converted from each source to the desired output frequency. This is referred to as noise folding.

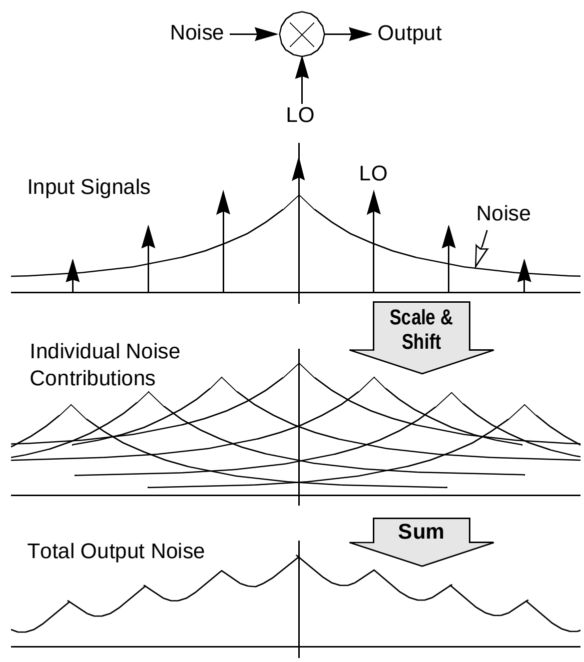

Fig. 10. How noise is moved around by a mixer. The noise is replicated and translated by each harmonic of the LO.

Periodic modulation of a stationary noise source, either from a periodic bias or from a periodically-varying signal path from the source to the output, results in cyclostationary noise at the output. In stationary noise, there is no correlation between noise at different frequencies. As can be seen from Figure 10, at frequencies separated by kf cyclostationary noise is correlated, where f is the modulation frequency and k is an integer [7]. The significance of this correlation will become apparent in Section VII-C.

VII. RF MEASUREMENTS

This section introduces several of the most common RF measurements with a description of how these measurements would be made using an RF simulator.

A. *Conversion Gain and Other Transfer Functions*

Conversion gain is the generalization of gain to periodically-varying circuits such as mixers. It is simply the small-signal gain through a mixer as a function of frequency. Typically, conversion gain refers to the transfer function from the desired input to the desired output. But there are many other transfer functions of interest, such as the gain from an undesired image or from an undesired input such as the power supply lines.

Remember that the output signal for a periodically-varying circuit such as a mixer may be at a different frequency than the input signal. The transfer functions must account for this frequency conversion. As described earlier, these circuits may have many images, and so for a single output frequency there may be many transfer functions from each input.

One measures a transfer function of a mixer by applying the LO, computing the steady-state response to the LO alone, linearizing the circuit about the LO, applying a small sinusoid, and performing one of the periodic small-signal analyses described in Section VI.

B. *AM and PM Conversion*

As shown in Figure 9, when a small sinusoid is applied to a periodically-driven circuit, the circuit responds by generating both the upper and lower sidebands for each harmonic. The sidebands act to modulate the harmonics, and the relationship between the sidebands determines the character of the modulation. In Figure 11 both the carrier and the sidebands are phasors. Assume that the sidebands are small relative to the carrier and that the circuit is driven at baseband with a small sinusoid with a frequency of f_m. The sideband phasors rotate around the end of the carrier phasor at a rate of f_m, with the upper sideband rotating one way and the lower rotating the other. The composite of the sideband phasors traces out an ellipse as shown in Figure 11b. However, if the two sidebands have identical amplitudes and their phase is such that they align when parallel to the carrier, the phase variations from each sideband cancels with the result being pure amplitude modulation (AM) as shown in Figure 11c. If instead the amplitudes are identical but the phases align when perpendicular to the carrier, then the amplitude variations cancel and the result is almost purely a phase modulation (PM) as shown in Figure 11d (assuming the sidebands are small). The DSB modulation shown in Figure 11b can be considered the combination of both AM and PM modulation.

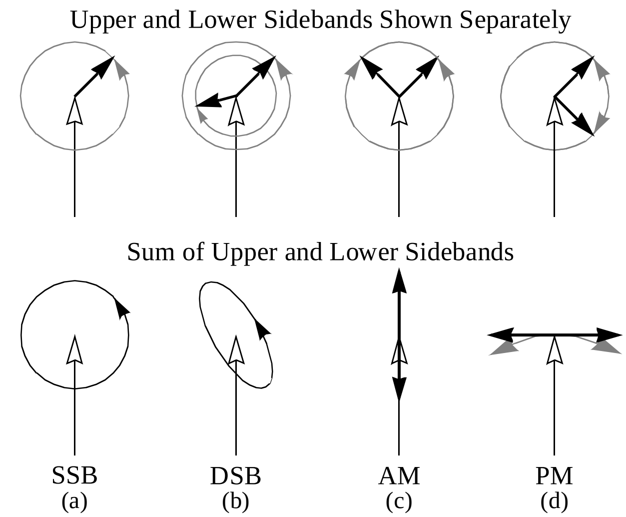

Fig. 11. How the amplitude and phase relationship between sidebands cause AM and PM variations in a carrier. The phasors with the hollow tips represents the carrier, the phasors with the solid tips represent the sidebands. The upper sideband rotates in the clockwise direction and the lower in the counterclockwise direction. The composite noise trajectory is shown below the individual components. a) Single-sideband modulation (only upper sideband). b) Arbitrary double-sideband modulation where there is no special relationship between the sidebands. c) Amplitude modulation (identical magnitudes and phase such that phasors point in same direction when parallel to carrier). d) Phase modulation (identical magnitudes and phase such that phasors point in same direction when perpendicular to carrier).

AM and PM conversion occurs either when a tone is injected at either baseband or at a sideband. The former is referred to as baseband to AM/PM conversion and the latter is SSB to AM/PM conversion. Both cases were demonstrated in the case of an oscillator by Razavi [23].

C. *AM and PM Noise*

As shown in Figure 10, periodically-driven circuits generate noise with correlated sidebands. And as shown in Figure 11, depending on the magnitude and phase of the transfer-function from the noise source to the output sideband, the noise at the output of the circuit can be AM noise, PM noise, or some combination. For example, oscillators almost exclusively generate PM noise near the carrier whereas noise on the control input to a variable gain amplifier results almost completely in AM noise at the output of the amplifier.

This ability to emphasize one type of noise over another is a characteristic of periodically-driven circuits. Linear time-invariant circuits driven by stationary noise sources can only produce *additive noise*, which can be decomposed into AM and PM noise, but there will always be equal amounts of both.

D. Noise Figure

Noise is a critical concern in receivers because of the small input signals. Typically designers characterize the noise of individual blocks using the noise figure of the block because it is relatively simple to combine the noise figure of cascaded blocks to determine the noise figure of the entire receiver [24]. The noise figure (*NF*) of a block is a measure of how much the signal-to-noise ratio (SNR) degrades as the signal passes through the block. It is defined as

$$NF = 10\log\frac{SNR_{\text{in}}}{SNR_{\text{out}}} \tag{15}$$

At the input of a receiver the *SNR* is defined as the signal power relative to the background noise power picked up by the antenna. From Figure 6 it is clear that a receiver is sensitive to noise at it input at each of its images. SNR_{in} only includes the noise power in the images where the input signal is found. In most communication systems, the input signal is found in a single sideband, and so single-side band (SSB) *NF* is employed. In this case, SNR_{in} includes only the noise power in that image associated with that sideband, though SNR_{out} does include the affect of the input noise from all images. Similarly, SNR_{out} excludes the noise generated in its load in the band of interest.

One computes the noise of a periodically-driven block by applying the LO, computing the steady-state response to the LO alone, linearizing the circuit about the LO, applying one of the periodic small-signal noise analyses mentioned in Section VI.

E. Intermodulation Distortion

Distortion is commonly measured in narrow-band circuits by applying two pure sinusoids with frequencies well within the bandwidth of the circuit (call these frequencies f_1 and f_2). The harmonics of these two frequencies would be outside the bandwidth of the circuit, however there are distortion products that fall at the frequencies $2f_1 - f_2$, $2f_2 - f_1$, $3f_1 - 2f_2$, $3f_2 - 2f_1$, etc. As shown in Figure 12 these frequencies should also be well within the bandwidth of the circuit and so can be used to measure accurately the *intermodulation distortion*, or *IMD*, produced by the circuit.

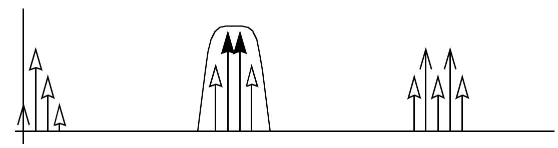

Fig. 12. A narrowband circuit driven with two closely spaced sinusoidal tones (—►) responds by generating harmonics (—>) and intermodulation (—▷) tones. Distortion of the output signal results because several of the odd-order intermodulation tone fall within the bandwidth of the circuit.

Compression and Intercept Points: At low frequencies, it is common to describe the distortion of a circuit by indicating the distortion in the output signal when driven by a sinusoid to achieve a certain output level. At high frequencies it is more common to characterize the distortion produced by a circuit in terms of a compression point or an intercept point. These metrics characterize the circuit rather than the signal, and as such it is not necessary to specify the signal level at which the circuit was characterized.

To understand the definition of a compression point and an intercept point, consider the output power of the fundamental and the 3$^{\text{rd}}$ order intermodulation product (IM_3) (either $2f_1 - f_2$ or $2f_2 - f_1$) produced by an RF circuit as a function of input power, as shown in Figure 13.

The 1 dB compression point is the point where the gain of the amplifier has dropped 1 dB from it small-signal asymptotic value. $\text{iCP}_{1\text{dB}}$ is the input power and $\text{oCP}_{1\text{dB}}$ is the output power that corresponds to the 1 dB compression point.

The third-order intercept point IP_3 is defined in terms of the power levels of IM_3 as extrapolated from their asymptotic small-signal behavior. When the input signal is small, a doubling of the input power results in a doubling fundamental output power and multiplies the output power of the third order products by 8 (2^3). Thus, the asymptotic slope of the fundamental is 1 dB/dB and the asymptotic slope of the third order products is 3 dB/dB. The third-order intercept point (IP_3) is where the asymptotes for the third harmonic and the fundamental cross. iIP_3 is the input power and oIP_3 is the output power corresponding to the intercept point.

ACPR and Spectral Regrowth: A very important issue when transmitting digitally modulated signals is adjacent channel power. It is important that a transmitter only emit power in its designated channel. Any power emitted in adjacent channels can interfere with the proper operation or nearby receivers that are attempting to receive signals from distant transmitters. As such, transmitters have strict

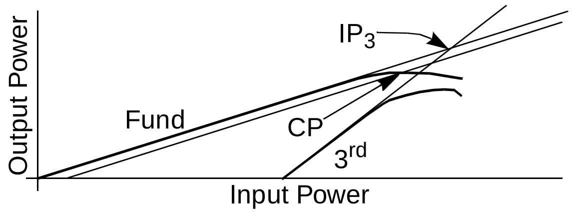

Fig. 13. The 1 dB compression point is the point where the output power of the fundamental crosses the line that represents the output power extrapolated from small-signal conditions minus 1 dB. The 3rd order intercept point is the point where the third-order term as extrapolated from small-signal conditions crosses the extrapolated power of the fundamental.

adjacent channel power specifications that they must satisfy. Unfortunately, this specification is very difficult to verify using simulation. Simple two-tone intermodulation tests are not representative of a digitally modulated signals. Instead, the transmission of a long pseudorandom sequence of symbols is simulated. The output spectrum is calculated from a sequence that typically contains between 1k and 4k symbols. The carrier frequencies are typically in the 1-5 GHz range and the symbols typically have a rate of 10-300 kHz. Such a simulation is clearly impractical for traditional transient analysis. Instead, the envelope methods briefly mentioned in Sections V-B and V-C are used. However, simulating a 1-4k symbol sequence still requires between 10k and 100k simulation points, each of which represents a harmonic balance or shooting method solve, and so even the envelope methods are extremely expensive for this type of simulation.

F. Phase Noise

One can apply the small-signal analyses of Section VI to oscillators to compute phase noise and sensitivity to small interfering signals such as those on the power supply. And as indicated in Sections VII-B and C, these analyses are able to properly account for frequency conversions and for the fact that response in the output manifests itself largely as changes in the phase of the output.

These analyses are small-signal analyses and assume that the circuit being analyzed does not respond in a nonlinear way to the small-signal inputs. However, (7) indicates that even small inputs can generate large changes in the phase if they are close in frequency to the output or to one of its harmonics. The output is a linear function of the phase only for small changes in the phase. If the phase changes by a significant fraction of a period, the response in a nonlinear way. It is this nonlinear response that causes the roll-off in L at very low frequencies that is given in (11). As a result, the small-signal analysis results do not predict the roll-off and so are inaccurate at frequencies very close to the carrier or its harmonics.

CONCLUSION

By exploiting the natural characteristics of RF circuits, RF simulators are able to efficiently perform simulations that were either impractical or impossible only a short time ago.

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